Annick Horiuchi, *Japanese Mathematics in the Edo Period (1600-1868)*, Basel: Birkhäuser 2010, xxvii+376 pp. Translated by Silke Wimmer-Zagier from the French original *Les Mathématiques japonaises à l'époque d'Edo 1600-1868*, Paris: Libraire Philosophique J. Vrin, 1994.

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Introduction

We have become used to looking to French scholarship for profound and comprehensive studies of history of mathematics in East Asia. It should thus not come as a surprise that the Birkhäuser Science Network Historical Studies chose the work of a France-based scholar, Professor Annick Horiuchi from Université Paris 7-Diderot, to fill the gap in general knowledge about Japanese mathematics. Originally published in French in 1994, Prof. Horiuchi's book was positively received by francophone historians of mathematics (Nagy 1995, Chemla 1996). Now it has been translated into English, without major updates, by Silke Wimmer-Zagier, with help from her husband, the mathematician Don Zagier.

A gap of more than fifteen years does not make an English translation of her text superfluous. For a start, very little has been published on Japanese mathematics in English, and nothing of comparable scope and depth since Yoshio Mikami and David E. Smith's book, which is now almost one hundred years old (Smith & Mikami 1914)! Although new research results have appeared in Japanese (especially Sato 2005, cf. the review of Horiuchi 2006) and other languages (such as the works of Prof. Xu Zelin in Chinese, summarised in Xu Zelin 2008) since the time of the French edition, it is safe to say that no revolution has occurred in our understanding of the main subject of this book, the careers and works of Seki Takakazu 関孝和 (?-1708) and Takebe Katahiro 建部賢弘 (1664-1739).

Despite its comprehensive-sounding title, Prof. Horiuchi's book is very tightly focused on these two mathematicians, who worked at a time "central to the development" of *wasan* 和算, and brought "essential contributions" to it (p. IX). Thematically, too, she stays close to the central topics of Seki's and Takebe's work, which are respectively "techniques for solving problems" (*kaidai no hō* 開題之法, a development of the Chinese *tianyuan* 天元 algebra) for Seki, and trigonometry for Takebe. The reader will find here nothing about magic squares and other problems also treated by the same mathematicians and their predecessors, which interested Smith & Mikamo (1914), and even the selection of mathematicians preceding Seki and Takebe is limited to those with a clear position in the intellectual ancestry of the two main characters.

This is, in the reviewer's opinion, the main strength of this book: it thoroughly analyses a few important problems concerning Seki's and Takebe's work, employing both close reading and minute understanding of the technical content of their writings, based on wide erudition about the cultural, social and intellectual context in which they worked. This includes the Chinese mathematical treatises known and studied in Japan in the seventeenth and eighteenth centuries, the conditions at the shogun's court, where both Seki and Takebe were employed for a time, and the changing attitudes of the shogunate to the technical knowledge of Western (especially Jesuit) origin in the early eighteenth century. Unfortunately, nothing is said about developments after the mid-eighteenth century, an era which produced a vigorous and wide-ranging mathematical culture. Here Smith and Mikami's work is still the only major source, supplemented recently by popular accounts written from mathematicians' perspective (Fukagawa & Rothman 2008).

Structure of the book: From the pioneers to Seki and Takebe

In the first four chapters, or roughly one quarter of the book, the author introduces seventeenth-century mathematical manuals which contributed most directly to later developments of *wasan*. She starts with the *Permanent Treatise* (*Jinkōki* 塵劫記, 1627) by Yoshida Mitsuyoshi 吉田光由 (1598-1672), the first Japanese mathematical manual using elements of the Chinese mathematical tradition. The second textbook, analysed in more detail, is Imamura Tomoaki's 今村知商 (fl. 1639-1660) *Jugai's Register* (*Jugairoku* 竪亥録, 1639), which served as an important transmitter of Chinese mathematical knowledge into new typically Japanese contexts.

Another stop on the journey to Seki and Takebe is the *Mathematical Platter* (*Sanso* 算俎, 1663) by Muramatsu Shikegyo 村松茂清 (1608-?). The *Sanso* reflected not only the thorough assimilation of the contents of the Chinese manuals, the *Compendium of Mathematical Methods* (*Suanfa tongzong*

算法統宗, 1592) by Cheng Dawei 程大位, and the more recently introduced *Initiation to Mathematics* (*Suanxue qimeng* 算學啟蒙, 1299) by Zhu Shijie 朱世杰, but also a new lively Japanese mathematical culture centred around the "bequeathed problems" *idai* 遺題. These open problems were first introduced as a challenge to readers at the end of the *Jinkōki* and became popular in later textbooks. This unique Japanese phenomenon stimulated mathematical research by introducing to classical problems of Chinese origin subtle variations that made the established solution algorithms unusable and necessitated more general considerations.

The introductory part ends with a brief discussion of the *Record of Old and New Mathematical Methods* (*Kokon sanpōki* 古今算法記, 1671) by Sawaguchi Kazuyuki 沢口一之, a book wholly devoted to the *tengen/tianyuan* method, which is thoroughly explained in this connection. Sawaguchi's treatise ended with 15 open "bequeathed problems" about geometrical figures and solids, which tested the limits of the original *tengen* and provided impetus to Seki's and Takebe's research.

The two great mathematicians are allocated two chapters each, one on their lives in their historical context, and one on their mathematical innovations. Seki served under the *bakufu* administration of the fifth Tokugawa shogun Tsunayoshi (reigned 1680-1709), whose policy initiatives favourable to the development of technical knowledge are related to Seki's scientific interests. Takebe's life and career are better documented than Seki's, as he served for more than twenty years in various functions to the shoguns Ienobu (reigned 1709-1712), Ietsuge (1713-1716) and Yoshimune (1716-1745). Both Seki and Takebe were involved in cartography and research on the calendar, and they benefitted from privileged access to Chinese books at court. Horiuchi is especially interested in Takebe's position vis-à-vis the reversal of the shoguns' policy on Jesuit works. These were summarily banned until 1720 as religious propaganda, but Yoshimune allowed the import and publication of Chinese scientific books containing material transmitted from the Jesuits. Mei Wending's 梅文鼎 (1633-1721) *Complete Works on Calendar and Mathematics* (*Lisuan quanshu* 曆算全書, published in China in 1723, transmitted to Japan in 1726) were especially important for Yoshimune's attempted, but ultimately failed, calendar reform. Although Takebe made several positive comments about the Western methods, Horiuchi concludes that he did not have direct influence on Yoshimune's decision, which stemmed from the shogun's genuine interest in mathematics and astronomy.

Although Seki and Takebe formed a close master-disciple relationship, Horiuchi draws a sharp distinction between their mathematical interests and styles. Seki is portrayed as the father of a uniquely Japanese theory of equations, presented in several books (mostly published after his death).

Seki's only monograph published during his lifetime is the *Mathematical Treatise Revealing the Hidden Meaning* (*Hatsubi sanpō* 発微算法, 1674), written in classical Chinese and consisting of solutions to the fifteen bequeathed problems from the *Kokon sanpōki*. The full method employed in these solutions was however only revealed by Takebe in the *Vernacular Commentary on the Endan* [analysis into geometrical blocks] *of the Hatsubi Sanpō* (*Hatsubi sanpō endan genkai* 発微算法演段諺解, 1685).

Shortly before 1685, Seki wrote a trilogy on solving "visible", "hidden" and "concealed" problems (*Kaikendai no hō* 解見題之法, *Kaiindai no hō* 解隱題之法, and *Kaifukudai no hō* 解伏題之法), which detailed the techniques necessary for transforming problems not amenable to the *tianyuan* method to ones that could be solved by it. Also in 1685, Seki wrote another trilogy devoted to problems, but with a different aim: rather than solving them, he wanted to identify problem statements that produced several or no valid solutions ("faulty problems" *byōdai* 病題) and suggest how to turn them into valid problems. Horiuchi focuses on the second of these treatises, the *Method of Extraction after Transformation* (*Kaihō honhen no hō* 開方翻變之法), widely discussed by Japanese historians as containing classic results of algebra (Horiuchi urges caution in this respect). She adds a short but upbeat assessment of the first and central treatise in this set, the *Method for Correcting Faulty Problems* (*Byōdai meichi no hō* 病題明致之法). Although little studied by historians and composed of case studies, it "bristles with ideas" on how to change the original equation to eliminate ambiguity (multiple roots).

Seki and the Takebe brothers (Katahiro and the younger Kataaki 賢明) together wrote the compendium *Accomplished Classic of Mathematics* (*Taisei sankei* 大成算経, written 1683-1700). Unlike Sato (2005), Horiuchi considers the authorship of this work to be unclear, and does not place it at the centre of either Seki or Takebe's contributions (cf. her still unconvinced review (2006) cited above). Moreover, Horiuchi argues that even the *Configurations of Extraction* (*Kaihō sanshiki* 開方算式), another work often ascribed to Seki, is no more than an extract from the *Accomplished Classic*, and probably a work of Takebe.

In contrast to Seki, portrayed as a "problem solver," the heart of Takebe's work is seen in trigonometry, reaching its high point in the period after 1716, when Seki was already dead. Since the two mathematicians cannot be so neatly separated, the chapter on Takebe's trigonometry nevertheless includes much about Seki's study of the circle, from the *Essential Summary of Mathematical Methods* (*Katsuyō sanpō* 括要算法, 1680), as well as from the *Accomplished Classic*. Takebe's arguably most famous book, the *Mathematical Classic of the Method of Appending (Tetsujutsu sankei* 綴術算経,

1722), built on these foundations a theory of trigonometric quantities calculated by infinite series. Horiuchi, as always, rightly subordinates these technical similarities to a thorough analysis of Takebe's abstract interests. The origin of his work, Horiuchi argues, lay in his conviction that mathematics is a study of natural objects. This conviction is expressed fully in the preface to *Tetsujutsu sankei*, but even more in Takebe's application of his results to calendrical science, analysed in the last pages of the book.

The main theme: *Wasan* between Chinese and Western mathematics

Although Horiuchi's work is not an encyclopaedic history of Japanese mathematics, it is very rich in the questions and topics it addresses. In the reading of this reviewer, the following three stand out as particularly persistent: the relation of *wasan* to the Chinese mathematical tradition, the internal and external motivations of its development, and the innovations and originality displayed in the work of Seki and Takebe.

Horiuchi's interest in the Chinese origins of *wasan* is motivated precisely by the search for originality and innovation. This attitude is directed against essentialist interpretations of *wasan* as an expression of Japanese national character, strongly present in Mikami's conception of the history of Japanese mathematics. Horiuchi writes in the preface that the notion of national character as something permanent and immutable "would arouse more suspicion today than in the years before the war" (p. XXIII), and pledges to base all explanations on particular historical, social and cultural facts. Moreover, Horiuchi takes exception to Mikami's discussion of the relation between Chinese and Japanese mathematics purely in terms of influence, which obscures the creative effort that goes into understanding a tradition and appropriating it into a new cultural context. On the other hand, by locating the starting point of Japanese researches in specific problems and algorithms taken from Chinese manuals, Horiuchi is able to avoid essentialist explanations of the later development of *wasan* and locate it "within mathematical practice itself, as it was inherited from China" (p. 61).

Horiuchi pays close attention to the ways in which this inherited practice was digested and rationalised in the seventeenth century textbooks. We find a repeated pattern here: methods which were originally classified in different chapters of Chinese books according to their chief area of application are regrouped in the Japanese manuals by their relatively abstract common points. For example, areas of fields and volumes of solids are treated as similar problems in the *Jugairoku*; systems of linear equations, traditionally falling into the Chinese category *fangcheng* 方程 ("rectangular arrays"; i.e. systems of linear equations), are solved in the chapter on "Excess and Deficit" (*eijiku* 盈朒) in Muramatsu's *Sanso*. There is surely a didactic motive behind these rationalisations, corresponding to the fact that authors of these textbooks were also successful teachers of mathematics. This is reminiscent of the thirteenth century teachers of mathematics in China, such as Yang Hui 楊輝. His rearrangement of the contents of the *Nine Chapters* in the last "Classification" (*zuanlei* 纂類) chapter of his *Detailed Explanations of the Nine Chapters* (*Xiangjie jiuzhang suanfa* 詳解九章算法) of 1261 shows similar pedagogical iconoclasm. Ironically, this work does not seem to have been known in Japan in the period covered by Horiuchi's study.

The Chinese influence on *wasan*, and on Seki in particular, has been studied by several authors since the original French publication of Horiuchi's book (Jochi 1994, Sato 1995, Martzloff 1998, Jochi 2000). These provide much more detail to corroborate the thesis that "Seki's style, at first sight new, can thus be considered as a reactivation of the style of the ancient Chinese treatises" (p. 166). Takebe's attitude to Chinese knowledge however became more ambiguous towards the end of his life: he held in high esteem the fragments of European mathematics and astronomy which he learned from Mei Wending's *Lisuan quanshu*, which leads Horiuchi to wonder whether this "may not have led him to call into question the traditional Chinese sciences of which *wasan* was the continuation" (p. 316).

What were the motors of the development of *wasan*? Horiuchi argues against the thesis, advanced by Mikami and repeated until today, that *wasan* was essentially an art practiced and perfected by leisurely samurai for its technical difficulty. Although this is to some extent the case in the nineteenth century, the mathematicians surveyed in this book are all practical men, studying and teaching mathematics as an aid to craft (Sawaguchi, the author of the important *Kokon sanpōki* of 1671, was mainly a carpenter), or to technical disciplines essential for the state (surveying and calendar). This practical orientation, however, does not exclude curiosity and the desire to excel in the discipline, which were strongly enhanced by the existence of competing schools and by the tradition of the "bequeathed problems." The development of the discipline was thus driven by an internal mechanism, facilitated by the social structures in which it was pursued (secretive, competitive schools, advertising their competence through published manuals and challenging each other through open problems). At the same time, the initial impulse for learning mathematics was external—the assumption that mathematics is useful knowledge. In this respect, Japan of the Edo period was no different from other early modern societies with a culture of mathematical research.

Horiuchi supports this view especially by her discussion of Takebe, the original main character of her 1990 dissertation. Takebe was an advisor on the calendar and other scientific questions and his work on trigonometry and approximations are directly linked to his duties. More controversially, Horiuchi also interprets Takebe's view of mathematical research, expressed in his preface to the *Tetusjutsu sankei*, as being "close to the physical sciences, where the objects being sought are to be found in Nature, waiting to be revealed" (p. 268). Takebe is shown to be saying that numbers, principles and rules are found in "the way of nature" (*shizen no michi* 自然之道). Although Horiuchi cautions against identifying this with European-style natural philosophy, she still thinks this objective view of mathematics, independent and often transcendent of the human mind, attests to Takebe's empirical philosophy of mathematics. Although he allowed both research from principles (i.e. deduction) and research from numbers (i.e. intuition supported by incomplete induction), Takebe seems to be more proud of his achievements obtained through the second method, which brought him closer to the part of mathematics "that we are unable to conceive." His method for the approximation of segments of arcs in the *Procedure of the Arc Following the Principle of the Circle* (*Enri kohaijutsu* 円理弧背術), using partial sums of infinite series, is a primary example of a formula derived by careful observation of several cases (pp. 287-288). Horiuchi identifies Takebe as a proponent of empirical mathematical research that is largely explained by his engagement with calendrical science (p. 271).

Finally, let us consider with Horiuchi the question of originality and the innovations introduced by Seki and Takebe in particular, and of the Japanese science of the Edo period more broadly. This is a traditional preoccupation of the history of science, and perhaps especially of the history of mathematics, concerned so often with priority, precedents, analogies, and ultimately with judging the value and attention-worthiness of historical individuals and their works. Horiuchi accepts it as an important question to be raised, but subtly distances herself from the most old-fashioned and reductive approaches to it. In the preface, Horiuchi recalls Mikami's rejection of comparisons "in terms of absolute worth" between Japanese and Western science. This, however, fell on the largely deaf ears of those who followed him, who still criticised Japanese mathematics for the absence of proof and other "defects." Horiuchi emphasises her ambition "to give an account of the development of mathematics in Japan, without bringing in criteria foreign to the Sino-Japanese tradition of mathematics" (p. XXV).

This issue comes up in two especially prominent discoveries traditionally attributed to Seki (determinants) and Takebe (infinite power series). Let us first note that Horiuchi refrains from applying the usual label "algebra" to the methods of *fangcheng*, *kaifang* ("opening up the side", i.e. root extraction), and *tianyuan*/*tengen* ("heavenly element"). Instead, there is a more narrowly defined category of "problem-solving techniques" (*kaidai* 開題), attested in the work of Seki. In a similar vein, determinants, although they arise in Seki's work, are better understood in the framework in which they appeared, rather than as an analogue of the Western concept of determinants. Seki was interested in diagonal products needed to eliminate an unknown from a system of polynomial equations, as laid out in the *Kaifukudai no hō*. Even though his interest transcended the practical needs of the problem being solved, too much excitement over determinants (evident in Mikami 1914, or even today in Hart 2011 for a related Chinese case) can be misleading, because other innovations by Seki were more important for the later development of *wasan*—especially his symbolic, literal notation of products (terms of polynomials). At the same time, Seki's tendency to arrive at general methods, manifested in the introduction of the literal notation, is also more in line with the long-term development tendencies of the whole Sino-Japanese tradition (p. 166).

Horiuchi explains Takebe's innovations by comparison with Newton's discovery of the binomial formula, the expansion of $(P + PQ)^{m/n}$ for fractional powers m/n. Takebe's series of sagittas and arcs was derived by a process similar to Newton's binomial expansions, starting from empirical observation of particular terms and modelled on previously known cases of infinite series. In this case, rather than comparing actual mathematical analogues, Horiuchi focuses on the general approach and finds that "the points of convergences prevail over the divergences" (p. 301). This also informs Horiuchi's assessment of the scientific initiatives of Takebe's patron, the shogun Yoshimune. Against dismissive statements such as Nakayama's that "[the] Japanese at this stage were still preoccupied with the exotic, new and utilitarian aspects of Western knowledge; they overlooked its deeper roots in a universal scientific method" (quoted on p. 230), Horiuchi suggests that given the selective nature of the Jesuit translations, and their complicated process of assimilation in Japan (i.e. without Euclidean geometry), the Japanese could not have learned much more about the scientific method than what they actually did: the essential role of mathematics in describing natural phenomena.

The Verdict

This reviewer cannot judge the place of Professor Horiuchi's work in the entirety of scholarship about *wasan*; what I can say, however, is that the book is an extremely attractive gateway into the historical questions surrounding the best-known period of *wasan*, one written from a modern methodological perspective. It must be said that it is not always easy to navigate the plethora of book titles and names appearing in various chapters, an aspect somewhat aggravated by the separation of mathematical and narrative sections. Parts of the historical narrative, especially concerning Seki, moreover seem rather incidental to the topics covered in the technical section, obviously the author's main interest. These technical sections are, in themselves, very clearly written and will surely engage any mathematically inclined reader; they include the essential prerequisites for understanding the underlying Chinese mathematics, written up with enviable lucidity and accuracy. A sinologist might be slightly inconvenienced by the absence of *kanji* in the main body of the book (moved to the comprehensive Glossary behind the text). But precisely as a sinologist, I was delighted and impressed by this book and its contents, with so many points of contact with and development of traditional Chinese mathematical knowledge.

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